

Pre-Calculus (4431) – Final Guess Paper

Q1. Arithmetic Progression (A.P.)

- (a) Insert 7 Arithmetic Means between 4 and 8.
 (b) If S_8 and S_9 are the sums of the first 8 and 9 terms of an A.P., and $S_9 = 63S_8$, $a_1 = 2$, find S_8 .
 (c) If three consecutive numbers in A.P. are increased by 1, 4, 15 respectively and the resulting numbers form a G.P., find the original numbers if their sum is 6.

Q2. Sets and Venn Diagrams

- (a) Using Venn diagrams, verify the following:
 (i) $(A - B) \cup B = A \cup B$ (ii) $A \cap B' = \emptyset \Leftrightarrow A \subseteq B$.

- (b) For $A = \{1, 2, 3, 4\}$, find the relations

(i) $R_1 = \{(x, y) \mid y + x = 5\}$ (ii) $R_1 = \{(x, y) \mid x + y < 5\}$.

State their domain and range and draw the graphs.

Q3. Matrices

- (a) Find the inverse of and verify that $AA^{-1} = I$.

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 3 & 1 \\ -8 & 5 & 5 \end{bmatrix}$$

- (b) If A and B are square matrices of the same order, prove:

(i) $(A + B)^2 = A^2 + 2AB + B^2$ (ii) $(A + B)(A - B) = A^2 - B^2$.

- (c) Solve the system using Cramer's Rule:

$$\begin{cases} x - 2y + 4z = 5, \\ 2x + y + 5z = 3, \\ x - 4y + 6z = -1. \end{cases}$$

Q4. Functions and Inverses

- (a) Find the inverse of $f(x) = 2x + 1$ and $f(x) = x^3$.

- (b) Find $(f \circ g)(x)$ if $f(x) = 3x^4 - 2x^2$ and $g(x) = \frac{2}{\sqrt{x}}$.

Q5. Quadratic Equations

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(a) Solve $4x^2 + 8x + 12 = 0$ by the Completing Square Method.

(b) Solve $x(x + 7) = (2x - 1)(x + 4)$.

Q6 Real Numbers and Domain/Range

(a) Define the real number system and its properties with respect to distribution and equality.

(b) Find the domain and range of:

(i) $f(x) = \sin x$ (ii) $g(x) = \frac{4}{8x-1}$.

Q7. Trigonometric Identities

Prove the given trigonometric identity (any two from past papers):

(i) $\sin^2 \theta + \cos^2 \theta = 1$ (ii) $1 + \tan^2 \theta = \sec^2 \theta$ (iii) $\sin 2\theta = 2 \sin \theta \cos \theta$.

Q8. Parametric Equations

If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, prove that

$$y \frac{dy}{dx} + x = 0.$$

Q9. Trigonometric Graphs

Draw the graphs of $y = \sin x$ and $y = \cos \left(\frac{x}{2}\right)$ for $-\pi \leq x \leq \pi$, showing amplitude and period.

Q10. Limits and Continuity

(a) Evaluate $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$.

(b) Discuss the continuity of at $x = 1$.

$$f(x) = \begin{cases} 3x - 1, & x < 1 \\ 4, & x = 1 \\ 2x, & x > 1 \end{cases}$$

Q11. Limits Involving e

Show that

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e.$$

Q12. Derivative by Definition

Find $f'(x)$ using the First Principle for each:

(a) $f(x) = \sqrt{x}$ (b) $f(x) = (x + c)^{1/3}$.

Q13. Differentiation Rules

Differentiate using the Product, Quotient, and Chain Rules:

(a) $y = x^3 e^{2x}$ (b) $y = \frac{ax+b}{cx+d}$.

Q14. Integration and Area

(a) Evaluate the definite integrals:

(i) $\int_0^{\frac{\pi}{2}} \cos t \, dt$ (ii) $\int_{-1}^5 |x - 3| \, dx$.

(b) Find the area between the x-axis and the curve $y = \sin 2x$ from $x = 0$ to $x = \frac{\pi}{3}$.

Q15. Miscellaneous and Applied

(a) A boat can row 10.6 km/h downstream and 6.8 km/h upstream. Find the speed of the boat in still water and the speed of the current.

(b) Prove that $\sqrt{5}$ and $\sqrt{7}$ are irrational numbers.

(c) Use **Synthetic Division** to find constants p, q if $(x + 1)$ and $(x - 2)$ are factors of $x^3 + px^2 + qx - 2$.

Preparation Tips

- Revise all **formulas**: Quadratic, Cramer's Rule, Trig Identities, Differentiation, Integration.
- Practice **2–3 sums** from each question type.